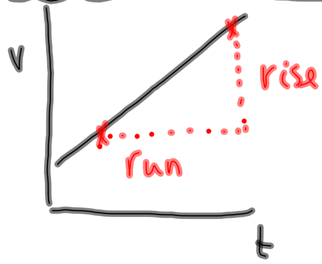
Acceleration & Velocity-Time Graphs

Constant Acceleration (Uniform)



slope = $\frac{\text{rise}}{\text{run}}$

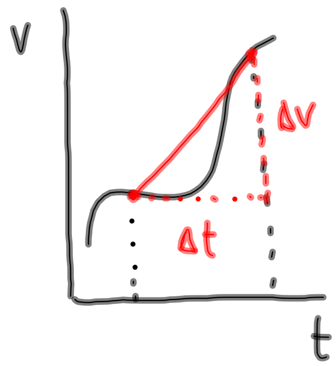
slope = $\frac{\Delta v}{\Delta t}$

slope = acceleration

$$a = \frac{\Delta v}{\Delta t}$$

This slope is constant since the graph is linear.

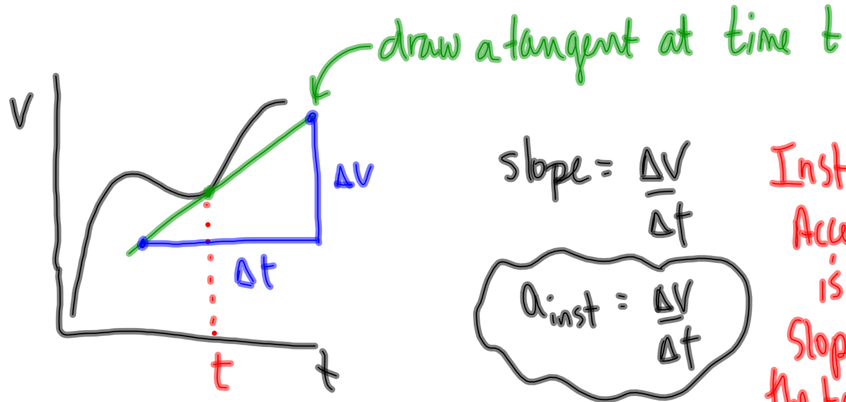
Non-Constant (Non-Uniform) Acceleration



slope = $\frac{\Delta v}{\Delta t}$

$$a_{ave} = \frac{\Delta v}{\Delta t}$$

Average Acceleration is the slope between two points on the v-t graph



slope = $\frac{\Delta v}{\Delta t}$

$$a_{inst} = \frac{\Delta v}{\Delta t}$$

Instantaneous Acceleration is the slope of the tangent drawn at t

- Eyeball the tangent
- use calculus!

The Acceleration Equation

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

where $\Delta\vec{v}$ is the velocity change (m/s)
$(\Delta\vec{v} = \vec{v}_f - \vec{v}_i)$

Δt is the time interval (s)

$\vec{a}$ is the acceleration (m/s²)

Example 1

A skier accelerates on her skis from 6m/s [forward] (v_i) to 15m/s [forward] (v_f) in 1.5s (Δt). What is her acceleration during this time?

let forward be +

$\vec{v}_i$ = 6m/s [forward] = +6m/s

$\vec{v}_f$ = 15m/s [forward] = +15m/s

Δt = 1.5s

a = ??

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{+15\text{m/s} - (+6\text{m/s})}{1.5\text{s}}$$

$$\vec{a} = \frac{+9\text{m/s}}{1.5\text{s}}$$

$$\vec{a} = +6\ \text{m/s}^2$$

$$\vec{a} = 6\ \text{m/s}^2\ [\text{forward}]$$

6ms^{-2}

IMPT ⇒ UNITS

$$\frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}} \div \text{s}$$

$$= \frac{\text{m}}{\text{s}} \times \frac{1}{\text{s}}$$

$$= \frac{\text{m}}{\text{s}^2}$$

Example 2

A skate boarder rolls down a hill with an average acceleration of (+) 0.40 m/s^2 [a]. He is on the hill for 4.8 s [Δt] and was going (+) 10.1 m/s [v_f] at the bottom of the hill. What was his velocity at the start?

(downhill) (downhill)

$\vec{a} = +0.40 m/s^2$

$\vec{v}_f = +10.1 m/s$

$\Delta t = 4.8 s$

$\vec{v}_i = ?$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a}\Delta t = \vec{v}_f - \vec{v}_i$$

$-v_f$ $-v_f$

$$\vec{a}\Delta t - \vec{v}_f = -\vec{v}_i$$

$$\vec{v}_i = \vec{v}_f - \vec{a}\Delta t$$

$$\vec{v}_i = +10.1 m/s - (+0.40 m/s^2)(4.8 s)$$

$$\vec{v}_i = +10.1 m/s - 1.92 m/s$$

Δv

$$\vec{v}_i = +8.18 m/s$$

$$\vec{v}_i \doteq +8.2 m/s$$

$$\vec{v}_i \doteq 8.2 m/s \text{ [downhill]}$$

UNITS:

$$\frac{m}{s^2} \cdot \frac{s}{1} = \frac{m}{s}$$

Example 3

A ball is dropped and falls until it reaches a velocity of 29.8 m/s [down]. How long was it falling?

(dropped ← $v_i = 0$) (falls ← implies $a = 9.81\,m/s^2$ [down] near the Earth's surface) (29.8 m/s [down] = v_f)

$\vec{v}_i = 0$

$\vec{a} = 9.81\,m/s^2\,[down] = -9.81\,m/s^2$ (− ← down)

$\vec{v}_f = 29.8\,m/s\,[down] = -29.8\,m/s$

$\Delta t = ?$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a}\,\Delta t = \Delta \vec{v}$$

$$\Delta t = \frac{\Delta \vec{v}}{\vec{a}}$$

$$\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

$$\Delta t = \frac{-29.8\,m/s - 0\,m/s}{-9.81\,m/s^2}$$

$$\Delta t = \frac{-29.8\,m/s}{-9.81\,m/s^2}$$

$$\Delta t = 3.037716616\ldots\,s$$

$$\boxed{\Delta t = 3.04\,s}$$

UNITS

$$\frac{m/s}{m/s^2} = \frac{m}{s} \div \frac{m}{s^2}$$

$$= \frac{m}{s} \cdot \frac{s^2}{m}$$

$$= s$$